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Original Article

A Simple Solution for Multi-Skilled Personnel Scheduling Problem

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Abstract - *Personnel scheduling for shift and task assignments has diverse applications and presents significant computational challenges, particularly in multi-skilled personnel scheduling problems. A recent methodology by Lalita and Murthy addresses large-scale shift and task scheduling but is limited to a homogeneous workforce. This article extends their solution to accommodate a heterogeneous workforce. Motivated by a real-world air traffic controller scheduling problem from a medium-sized airport, this article proposes a streamlined and effective solution to the multi-skilled personnel scheduling problem using mixed integer linear programming formulations. This approach enables large-scale, multi-skilled personnel scheduling problems to be solved efficiently in a short time. Two key objectives for the air traffic controller scheduling problem that inspired this work are addressed: assessing staffing needs for shift scheduling and creating monthly rosters.*

Keywords - *Multi-Skilled personnel scheduling, Air traffic control, Optimization, Mixed integer linear programming formulations.*

1. Introduction

Multi-skilled Personnel Scheduling Problems (MSPSPs) arise when organizations seek to efficiently allocate staff with multiple skills to meet operational needs across diverse tasks or shifts. These problems are prevalent in healthcare, manufacturing, and service industries, where resource constraints, complex task requirements, and fluctuating demand necessitate versatile workforce deployment. Air traffic controller scheduling is one important area under MSPSP. Despite the wide applicability of MSPSPs, existing research often focuses on general scheduling methods without addressing domain-specific challenges. For instance, in air traffic controller scheduling, factors like stringent regulatory requirements, varying workload intensities, and fatigue management complicate the scheduling process, yet these factors are underexplored in optimization literature. This research seeks to fill this gap by focusing on domain-specific characteristics of MSPSPs in air traffic management. MSPSP problems are computationally challenging, and various studies have investigated optimization techniques to address the problems, often employing integer programming, constraint programming, or heuristic approaches to enhance solution efficiency. This article proposes a simple and effective solution procedure for MSPSP using Mixed Integer Linear Programming (MILP) formulations. The procedure has evolved from addressing the problem of developing a solution for scheduling the Air Traffic Controllers (ATCs) of a moderate-sized airport. The main objective of the procedure is twofold: (i) to determine the staff requirements for shift scheduling of ATCs and (ii) to determine the monthly rosters. While the methods proposed in this article primarily focus on ATCs' scheduling, they apply to generic multi-skilled personnel scheduling problems. Lalita and Murthy [20] developed an elegant task and shift scheduling procedure with single-skilled operators. This article builds upon their work by extending the approach to accommodate multi-skilled operators. It also offers a streamlined and efficient method for tackling the MSPSP, focusing on ATC scheduling. It proposes simple MILP formulations as a more accessible and practical alternative to the complex models commonly found in the literature. The organization of this article is as follows. Section 2 describes the generic MSPSP problem and details ATC scheduling. Section 3 presents a literature review of MSPSP and the scheduling of ATCs. Section 4 presents the mathematical formulations for solving the problems. Section 5 presents the results of applying the methodology to the data on the airport for which the problem was taken up. Section 6 discusses the scope for future research. Section 7 concludes the article with a summary and some observations.

2. Problem Description

Personnel scheduling is done at strategic, tactical, operational and real-time planning levels. Strategic and tactical planning focuses on determining the staff requirements at a broad level and the optimal allocation of planned or available resources, typically over long periods. The procedure developed in this article is useful for strategic and tactical level planning. To simplify the problem description, the planning horizon is set to a monthly scale, consistent with the ATC scheduling issue that motivated this research. Subsection 2.2 provides a more detailed description of the MSPSP as encountered in the context of ATC scheduling.

2.1. MSPSP Scheduling Problem

A set of tasks are to be performed repeatedly daily over a planning horizon of a specified number of consecutive days. As mentioned earlier, a planning horizon of 30 consecutive days is considered. Tasks require certain skills and can be carried out by only those operators who possess the required skills specific to the tasks. Each day is divided into shifts, and the operators work in shifts. A variety of restrictions are imposed on the working conditions of the operators. These conditions may include the number of consecutive days of work, time gap between consecutive working shifts, number of days off, maintaining qualifications to perform the tasks by the operators for which they are qualified, etc. The objectives of MSPSP depend on the field of application or the industry in which it is applied. Each field has unique objectives, typically shaped by the nature of the work, resource constraints, and desired outcomes. Some common objectives in MSPSPs across various industries include reducing labor costs, maximizing service quality or customer satisfaction, distributing workload evenly to prevent fatigue, improving job satisfaction, reducing turnover, utilizing personnel skills optimally, ensuring sufficient coverage per shift and compliance with labor laws, union agreements, and safety standards, maximizing flexibility, reliability and adaptability of the schedules, etc.

2.2. ATC Scheduling

Air traffic controllers work in specialized facilities designed to monitor and manage the safe movement of aircraft. Depending on their functions, the facilities are divided into units and subunits requiring hierarchical skill levels. ATCs undergo extensive initial and on-the-job training to obtain licenses for working in different units. Below is a brief description of the Units/Subunits and their functions for a medium-sized airport.

2.2.1. Briefing Unit

Provide essential pre-flight information to pilots and flight dispatchers, including weather data, Notices to Airmen, and operational updates, ensuring safe and efficient flight planning. Help pilots make informed decisions before departure, minimizing potential in-flight issues.

2.2.2. Control Unit

This unit has three subunits - Control Delivery Unit (CLD), Surface Movement Control (SMC) unit and Tower Control Unit (TWR). CLD is responsible for issuing clearances and instructions to aircraft on the ground, coordinating their movement safely from gates to runways for takeoff. SMC manages and coordinates the safe and efficient movement of aircraft and vehicles on the airport’s taxiways and aprons, minimizing ground congestion and preventing collisions. TWR manages aircraft movements on runways and in the immediate airspace surrounding the airport, ensuring safe takeoffs, landings, and transitions to and from ground control.

2.2.3. Approach Surveillance Unit (APP)

The Approach Surveillance Unit manages both arriving and departing aircraft within terminal airspace, providing radar-based instructions to ensure safe separation, efficient sequencing, and smooth transitions to and from enroute sectors. This unit coordinates aircraft spacing and descent for arrivals and monitors departures as they climb, maintaining orderly flow and minimizing delays. This unit has two subunits - APP-ARR for arrivals and AAP-DEP for departures.

2.2.4. Area Control Unit (ACC)

The Area Control Unit (ACC) manages enroute air traffic, ensuring safe separation, efficient traffic flow, and coordination across large regions. It provides weather updates, handles potential conflicts, and assists with emergencies, supporting safe and efficient flight operations. This has two subunits - ACC Planning and ACC-Surveillance. The planning Unit organizes flight paths and coordinates air traffic to optimize flow and prevent congestion. At the same time, the Surveillance Unit monitors aircraft positions and ensures safe separation, swiftly identifying and addressing potential conflicts.

2.2.5. Work Supervisory Officer (WSO)

Oversee the operational activities and personnel in the control unit, ensuring safe, efficient, and orderly air traffic management. Units and subunits within departments require qualified ATCs to carry out operations. ATCs obtain licenses through training and experience, allowing them to work across various units and subunits. For this article, ATCs are categorized into five levels. All the supporting staff require no license, so their license level will be treated as zero. Also, ATCs working in the Briefing unit need no license. Hence, their license level is also zero. License levels for other units/subunits are 1 for ACU, 2 for APP, 3 for ACC-Planning and 4 for ACC-Surveillance and WSO. ATCs with higher level licenses are qualified for all units/subunits that require lower-level licenses. Tasks performed by ATCs are critical and involve high stakes. They require continuous concentration, rapid response, and detailed coordination to ensure the safe movement of aircraft. Due to the demanding nature of their work, ATCs are mandated to take regular breaks to manage stress and prevent fatigue, as even small lapses in focus can have profound safety implications.

Table 1. Tasks and their requriments

Task code	Task description	Minimum Skill code required	No. of ATCs required	
			Day shifts	Night Shift
1	BU operations	1	2	3
2	ACU primary operations	2	5	7
3	ACU supporting operations	1	1	2
4	ASU primary operations	3	3	4
5	ASU supporting operations	1	1	2
6	Area Planning primary operations	4	2	3
7	Area Planning supporting operations	1	2	3
8	Area Surveillance operations	5	1	2
9	Work supervisory Officer	5	1	2
License level Skill code 1.				

Further, the ATCs are assisted by supporting staff. The supporting staff, such as administrative personnel, data assistants, or technical support, play essential roles in maintaining the flow of operations but typically engage in less intense and non-critical tasks. Consequently, supporting staff members typically do not require structured breaks, as their tasks may be less time-sensitive or mentally demanding. For the ATC scheduling problem considered in this article, ATCs work in three shifts - morning and afternoon shifts (these two shall be referred to as day shifts) and night shifts. Day shifts are 8 hours long, and night shifts are 12 hours long. Considering the restrictions discussed above, staff requirements are determined for all the tasks of the units and subunits. Table 1 summarizes these requirements. Skill code is defined as license level plus one.

Two problems are considered in this article:

- Determine the number of ATCs at different levels to perform the tasks over the planning horizon and
- Determine the actual assignment of ATCs to tasks of units/subunits.

3. Literature Review

Early research in multi-skilled personnel scheduling, focused on applying mathematical programming and workforce planning models to sectors like healthcare and service industries, can be found in Batta et al., Loucks and Jacobs, Cai and Li [10] and Batta et al.. These foundational studies explored the complexities of matching tasks with workers' skills and managing shift assignments to ensure efficiency. Managing multi-skilled personnel involves balancing various skill sets across tasks and shifts, leading to more intricate scheduling challenges. Ernst et al. and Van den Bergh et al. [22] review the complexities of multi-skilled scheduling, highlighting the increased computational demands due to the greater flexibility required for skill-based allocations. Several authors have explored versatile solution methods. Burke et al. [5] introduced hybrid metaheuristic approaches, combining genetic algorithms and simulated annealing, to tackle scheduling problems in manufacturing, highlighting the importance of optimization in multi-skilled workforce management. In more recent work, researchers have continued to advance the field with more flexible and dynamic approaches. For example, Caglar Gencosman et al. [9] applied constraint programming to address fluctuating task demands. Gaudioso and Hadid [12] extended these concepts to transportation and logistics, using hybrid optimization models to improve scheduling in complex environments. These studies demonstrate the growing sophistication of MSPSP solutions, reflecting the increasing need for adaptive and efficient scheduling mechanisms in industries with diverse workforce skill sets. When tasks are pre-fixed, the shift and task scheduling problem is reduced to shift scheduling. Even the single-skilled shift scheduling problems are complex [7, 18]. High scheduling flexibility results in a huge number of personnel schedules. Mathematical programming formulations for solving staff scheduling problems are mostly based on the set-covering formulation of Dantzig [11]. As the set covering formulation requires all personnel schedules, decomposition and column generation techniques through implicit formulations are commonly used to handle the situation.

Implicit formulations are developed for several applications (see Thompson and Pullman [21], Thompson [20], Jarrah et al., Jacobs and Brusco [14], Aykin [2], Brunner et al. [6], and Sungur et al. [19]). The problem remains complex as the implicit formulations often result in a large number of constraints (see Bellenguez-Morineau and N´eron [3] and Brunner et al. [6]). The decomposition technique is used to break down the problem into stages to reduce the size of the problem (see Jarrah et al., Alfares [1], Stolletz [18] and Brunner and Stolletz [7]). Also, see Brucker et al. [5] to discuss models and complexities in personnel scheduling problems. Volland et al. [23] propose a procedure for the Integrated Shift and Task Scheduling Problem (ISTSP) using MILP formulations. This procedure involves solving a series of subproblems iteratively using the column generation technique. As an alternative, Lalita and Murthy [16] propose an elegant procedure to solve ISTSP that involves solving just two ILPs (see Lalita and Murthy [16]). However, these procedures are developed for single-skilled personnel scheduling problems [23]. The main contribution of this article is to extend the solution procedure proposed by Lalita and Murthy [16] to multi-skilled personnel scheduling problems. The Air Traffic Control scheduling problem is a specific application of the MSPSP. Integer linear programming and mixed-integer linear programming models are commonly applied to optimize air traffic controller scheduling by balancing staffing needs with constraints like fatigue, sector demands, and shift preferences. Wang and Ke developed a MILP model to reduce peak fatigue in Taiwans ATC scheduling, showing significant improvements in fatigue management. Similarly, Stojadinovi [17] used an ILP approach with SAT-based solvers to integrate shift preferences and qualifications for Serbian ATC staff, achieving results over an extended planning horizon. Josefsson et al. [15] implemented a MILP model in Sweden, reducing ATC staff requirements by automating remote tower staffing across multiple airports. Guo and Bard [13] investigate the air traffic controller scheduling problem by developing a 2-phase algorithm that efficiently assigns shifts and breaks to minimize staffing costs while meeting demand coverage and regulatory constraints. Testing their approach on FAA-defined and more significant instances, they demonstrate that the proposed method significantly improves upon existing schedules across key performance metrics, with a parametric analysis revealing how staffing levels impact solution quality. This article offers a streamlined and effective approach to tackling the MSPSP, specifically ATC scheduling, by introducing simplified MILP formulations as an alternative to the intricate models in the existing literature.

4. Formulation

This section presents formulations and a procedure for solving a generic multi-skilled personnel scheduling problem. Subsection 4.1 presents a multi-skilled personnel scheduling problem framework and the notation. The basic idea is to solve the problem in two stages. The first stage determines the personnel schedules for each task separately with the minimum number of operators. The procedure and the formulations for this problem are presented in Subsection 4.2. Using the minimum number of operators for each task thus obtained, an assignment problem is solved to determine a solution to the main problem of scheduling operators for the multi-skilled personnel scheduling problem. This assignment problem, the second stage problem, is discussed in Subsection 4.3.

4.1. Multi-skilled Personnel Scheduling Problem

A multi-skilled personnel scheduling problem may be described as follows. A set of tasks, each requiring a subset of different skills from operators, is carried out repeatedly over a period. This period, known as the planning horizon, is divided into discrete time intervals. For the problem considered in this article, the planning horizon is a month divided into days. The number of operators required to perform each task during each time period and the skills required to perform the task are known. The operators work in shifts to perform the tasks following mandatory scheduling regulations, including shift timings and breaks (minimum time gap between successive shifts and the days off). The scheduling problem is determining a roster for the operators to minimize an associated cost.

Let T = {1, 2, . . . , T} be a set of tasks to be performed during each time period of a planning horizon H = {1, 2, . . . , N}. Let S = {1, 2, . . . , S} be the set of skills of the operators who perform the tasks. Let P_s, $s \in$ S, be the set of operators with skill s. Let p_s be the number of operators available with skill s, i.e., $|P_s| = p_s$. Let r_{ht} be the number of operators required to perform task t for the time period h.

4.2. Scheduling ATCs for a Given Task

This section describes the complete procedure for scheduling the monthly rosters of ATCs for a specific task. ATC roster scheduling is done monthly. As there are three shifts - morning, afternoon and night - in a day, the total number of shifts in 30 days is 90. Number the shifts as 1 through 90 in chronological order. Take the planning horizon as $\mathcal{H}$ = {1, 2, . . . , 90}. Night shifts are longer than day shifts (morning and afternoon shifts) and need more personnel. Shift numbers of the form $3k + 1,\ k = 0, 1, 2, \dots, 29$ correspond to morning shifts, those which are multiples of 3 correspond to night shifts, and the rest correspond to afternoon shifts. ATCs are identified with operators in this problem. Tasks are identified with Unit/Subunit operations.

There are 9 tasks, coded as 1 through 9. See Table 1 for the list of tasks and the corresponding codes. The problem of scheduling ATCs for a particular task, say task t, requires two inputs: (i) the number of operators required to perform the task in each time period h, namely, r_{ht}, and (ii) the shift patterns of the operators. With these inputs, the task scheduling operator problem can be solved using the procedure developed by Lalita and Murthy [16]. This procedure will be briefly outlined now (see [16] for complete procedure). Understanding the shift patterns is necessary and convenient for describing the procedure. Since this article's main objective is to solve the ATC scheduling problem, the procedure will be described with specific reference to ATC scheduling.

Under the present system, each ATC serves one shift daily for three consecutive days, followed by two rest days. In three consecutive working days, an ATC may be assigned any combination of morning, afternoon and night shifts. Denoting the morning shift by 1, the afternoon shift by 2 and the night shift by 3, there are 27 possible combinations. Shift pattern abc stands for working in shift a on the first day, in shift b on the second day and shift c on the third day, $a, b, c \in \{1, 2, 3\}$, taking breaks on the 4th and fifth days. Some of these combinations may be inadmissible. For instance, shift patterns 131, 231, 231, 331, 312, and 313 are directly inadmissible as these involve continuous duty for 18 hours. Dropping these six, Figure 1 presents 21 possible shift patterns. For formulation, these 21 patterns are coded as 1 through 21.

Shift Patterns

Pattern	111	112	113	121	122	123	132	133	211	212	213	221	222	223	232	233	321	322	323	332	333
Code	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Shift pattern representation	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
	1	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	1	1	0	0	0	0	0	1	1	1	0	0	1	1	1	0	0
	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0
	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0
	0	1	0	0	1	0	1	0	0	1	0	0	1	0	1	0	0	1	0	1	0
	0	0	1	0	0	1	0	1	0	0	1	0	0	1	0	1	0	0	1	0	1

Fig. 1 Shift patterns for ATC

Each shift pattern has a 9-tuple binary vector associated with it (presented in Figure 1). Let $b^i = (b_{i1}, b_{i2}, \dots, b_{i9})$ denote the binary vector corresponding to shift pattern i, $i = 1, 2, \dots, 21$. These patterns specify the working shifts for three consecutive days. For example, an operator who is assigned a shift pattern $b^9 = (0, 1, 0, 0, 1, 0, 1, 0, 0)$ for a given day works in the afternoon on the first day, in the morning shift on the next day and in the morning shift on the third day. With the shift patterns defined above, the scheduling problem can be solved using the procedure developed by Lalita and Murthy [16]. This procedure will be briefly outlined now (see [16] for complete procedure). For brevity of notation, let $q = (q_1, q_2, \dots, q_N)$, where $q_h = r_{ht}$. The ATC shift scheduling problem has two inputs: q and the 21 shift patterns described above. The procedure of Lalita and Murthy [16] also involves solving two optimization problems: Optimization Problem 1 and Optimization Problem 2. These are described below.

4.2.1. Optimization Problem 1

A shift schedule is a two-tuple (abc, d) that stands for an ATC starting the work on day d with shift pattern abc. For instance, an ATC who is assigned shift schedule (221, 5), or equivalently (12, 5) using the shift pattern code 12 for 221, works in the afternoon shifts of days 5 and 6 (the corresponding time periods of $\mathcal{H}$ are 14 and 17), in the morning shift of day 7 (corresponding time period 19) and takes break on days 8 and 9 (corresponding time periods 22 to 27). The decision variables are defined by x_{ij} , the number of shift schedules (i, j), $i = 1, \dots, 21, j = 1, 2, \dots, 30$ to be assigned to the operators. These decisions result in the number of ATCs available for each $h \in \mathcal{H}$. Let $\overline{x} = (\overline{x}_1, \overline{x}_2, \dots, \overline{x}_N)$ denote the resulting vector, where $\overline{x}_h$ is the number of ATCs available for the time period h. The formula for $\overline{x}_h$ is given by

$$\overline{x}_h = \sum_{i=1}^{21}(x_{i29} b_{i(h+6)} + x_{i30} b_{i(h+3)} + x_{i1} b_{ih}), \quad h = 1, 2, 3, \tag{1}$$

$$\overline{x}_h = \sum_{i=1}^{21}(x_{i30} b_{i(h+3)} + x_{i1} b_{ih} + x_{i2} b_{i(h-3)}), \quad h = 4, 5, 6, \tag{2}$$

$$\overline{x}_h = \sum_{i=1}^{21} x_{i1} b_{ih} + \sum_{i=1}^{21} x_{i2} b_{i(h-3)} + \sum_{i=1}^{21} x_{i3} b_{i(h-6)}, \quad h = 7, 8, 9, \tag{3}$$

and

$$\overline{x}_{3d+h} = \sum_{i=1}^{21} x_{i(d-2)} b_{i(h+6)} + \sum_{i=1}^{21} x_{i(d-1)} b_{i(h+3)} + \sum_{i=1}^{21} x_{id} b_{ih}, \quad d = 3,4,\dots,29, \quad h = 1, 2, 3. \tag{4}$$

Equation 1 includes the terms x_{i29} and x_{i30} due to the roster. This is because those who start work on 29^{th} and on 30^{th}, are available for $1st$. A similar explanation holds good for Equations 2 to 4. Thus, $\overline{x}_h$s are linear functions of the decision variables x_{ij}s. The Optimization Problem 1 (OP1) is given by

Optimization Problem 1 (OP1)

$$\text{Minimize} \quad \sum_{i=1}^{21} \sum_{j=1}^{30} x_{ij} \tag{5}$$

subject to

$$\overline{x}_h \geq q_h, \; h = 1,2, \dots, N, \tag{6}$$

x_{ij}s are nonnegative integers, $i = 1,2, \dots ,21$, $j = 1,2, \dots ,30$.

The objective function in Equation 5 is the total number of shift schedules. The constraints in Equation 6 ensure that the required number of ATCs will be available in each time period h.

4.2.2. Optimization Problem 2

This problem takes the optimal solution of OP1, the optimal shift schedules obtained by solving OP1, and assigns them to ATCs obeying the scheduling rules/guidelines. It is much simpler to solve than those discussed in Lalita and Murthy [20]. This is because of how the shift schedules are defined. The method is described in the next paragraph.

Let $\{x_{ij}^o : i = 1, 2, \dots, 21, \; j = 1, 2, \dots, 30\}$ be an optimal solution of OP1. List all shift schedules (i, j) for which $x_{ij}^o > 0$. Let $\tau = \sum_{i,j} x_{ij}^o$ be the total number of shift schedules used to meet the demand requirements. Arrange the τ shift schedules in the ascending order of their day coordinate and denote them by $U_1, U_2, \dots, U_\tau$ so that if $k < k'$, then $j \leq j'$, where $U_k = (i, j)$ and $U_{k'} = (i', j')$. The optimization problem presented in Equations 7 to 11 is solved to find the minimum number of ATCs to handle the τ shift schedules. Label the ATCs as $1, 2, \dots, W$, where W is some upper limit on the number of possible ATCs. Let u_{ij} be the indicator variable, which is 1 if ATC i is assigned a shift schedule U_j.

Say that a pair of shift schedules $U_k = (i, j)$ and $U_{k'} = (i', j')$ are overlapping if the difference between *j* and *j'* is less than 5. For example, $(i, 8)$ and $(i', 12)$ are overlapping, but $(i, 8)$ and $(i', 14)$ are not. Taking the roster into consideration, $(i, 28)$ and $(i', 1)$ are overlapping. Let $\mathcal{P}$ be the set of all (k, k') for which U_k and $U_{k'}$ are overlapping. A feasible assignment cannot assign both of shift schedules U_k and $U_{k'}$ to the same ATC if they overlap. This is ensured by imposing the constraint $u_{ik} + u_{ik'} \leq 1$ if $(k, k') \in \mathcal{P}$. The Optimization Problem 2 (OP2) described below is to assign all τ shift schedules to ATCs so that each ATC gets at most 6 shift schedules and overlapping shift schedules are not assigned to the same operator. The objective of OP2 is to minimize the number of ATCs.

Optimization Problem 2 (OP2)

$$\text{Minimize} \quad t \tag{7}$$

subject to

$$\sum_{j=1}^{\tau} u_{ij} \leq 6, \quad i = 1, 2, \dots, W, \tag{8}$$

$$\sum_{i=1}^{W} u_{ij} = 1, \quad j = 1, 2, \dots, \tau, \tag{9}$$

$$u_{ik} + u_{ik'} \leq 1, \quad \text{for all } (k, k') \in \mathcal{P}, \tag{10}$$

$$\sum_{i=1}^{W} i u_{ij} \leq t, \quad j = 1, 2, \dots, \tau, \tag{11}$$

$$t \geq 0,, \quad i = 1, 2, \dots, W, \quad j = 1, 2, \dots, \tau.$$

Constraints in Equation 8 ensure that no ATC is assigned more than six shift schedules. Constraints in Equation 9 ensure that all the τ shift schedules are assigned. Constraints in Equation 10 are explained in the paragraph before the above formulation. The expression on the left-hand side of Equation 11 is the ATC number to whom shift schedule j is assigned. The decision variable t is a dummy variable used to minimize the number of ATCs. This constraint and the objective will ensure that the formulation finds the smallest number of ATCs to handle all the τ shift schedules. The assignment problem clearly has a feasible solution. Solving this problem yields a solution with a minimum number of ATCs.

Table 2. Summary of results for all the 9 tasts

Groups	Tasks								
	1	2	3	4	5	6	7	8	9
1	8	24	4	13	4	13	13	4	4
2	6	26	6	15	6	9	9	6	6
3	6	15	9	9	9	7	7	9	9
4	4	20	1	11	1	9	9	1	1
5	8	23	10	14	10	8	8	10	10
No. of shift schedules	32	108	30	62	30	46	46	30	30
Minimum number of ATCs	8	22	6	13	6	9	9	6	6

4.3. The ATC Assignment Problem

The minimum number of ATCs required for each task is determined using the above procedure. Let R_t be the number of ATCs required for task t, $t = 1, 2, \dots, T$. The inputs R_ts is obtained by solving OP1 and OP2 for each task. To complete the ATC scheduling problem, it suffices to group the available ATCs into T groups so that t^{th} group has R_t ATCs who are qualified to perform task t, $t = 1, 2, \dots, T$. This problem is solved using the formulation below. Let M be the total number of ATCs available. A known input is whether a particular ATC is qualified to perform a task. Define the incidence matrix A whose $(i, t)^{th}$ element is a_{it}, where $a_{it} = 1$ if ATC i is qualified to perform task t, and $a_{it} = 0$ otherwise. Let v_{it} be the indicator variable, which is 1 if ATC i is assigned to task t. Let c_{it} be the cost of assigning task t to ATC i. Consider the following problem.

Optimization Problem 3 (OP3)

$$\text{Minimize} \quad \sum_{i=1}^{M} \sum_{t=1}^{T} c_{it} a_{it} v_{it} \tag{12}$$

subject to

$$\sum_{i=1}^{M} a_{it} v_{it} \geq R_t, \quad \text{for} \quad t \in \mathcal{T}, \tag{13}$$

$$v_{it} \in \{0, 1\}, \quad i = 1, 2, \dots, M, \text{ and } t \in \mathcal{T}.$$

The objective function in Equation 12 is the total cost of assigning ATCs to the tasks. The constraints in Equation 13 ensure adequate operators are assigned to perform the tasks. To start with, OP3 can be solved with a large positive integer M instead of using the information on the actual ATCs available.

Thus, the solution can be used to check the adequacy of ATCs level-wise. This analysis is helpful for a general multi-skilled personnel scheduling problem where the ATC may be hired temporarily. The model can also be deployed when some ATCs are regular employees, and others are available temporarily or over time.

		Tasks									ATC Availability	Costs (Years)
		1	2	3	4	5	6	7	8	9		
Skill Level (= License code + 1)	1	1	0	1	0	1	0	1	0	0	21	2
	2	1	1	1	0	1	0	1	0	0	41	3
	3	1	1	1	1	1	0	1	0	0	1	5
	4	1	1	1	1	1	1	1	0	0	4	7
	5	1	1	1	1	1	1	1	1	1	33	10
ATC Requirements	Day Shifts	2	5	1	3	1	2	2	1	1		
	Night shift	2	7	2	4	2	3	3	2	2		

Fig. 2 Data for a moderate size airport

5. Application to a Live Instance

The methodology described in the previous section will be applied to solve an ATC scheduling problem with a monthly planning horizon. Data presented in Figure 2 pertain to a moderate size airport. It provides the availability of ATCs license wise (License code is equal to skill code minus one), the number of ATCs required for the three shifts, and the necessary inputs for the incidence matrix. The costs column is the average number of years an ATC takes to acquire a license. These costs, which are functions of the level of qualification and the task, can be used to derive an objective function.

A more general cost structure of the form c_{it} may also be considered. The following subsections present the results of solving different problems discussed in the previous section. There are 9 tasks in this problem, and the required inputs are in Table 1. The shift patterns are in Figure 1. Task requirements for tasks 3, 5, 8 and 9 are identical (see Figure 2). Therefore, it suffices to solve one of these problems. Similarly, task requirements for tasks 6 and 7 are identical, and it suffices to solve one of them. The results of solving the problems using OP1, OP2 and OP3 are presented below.

5.1. Results for OP1 and OP2

Applying OP1 and OP2 to the data results in shift schedules for each task (separately). The results for task 1 are summarized in Figure 3. For this task, OP1 resulted in 32 shift schedules, namely, 1, 16, 17, 20 and 21. The number of times each of these shift schedules is used, the frequency, is shown in the figure. Using OP2, these shift schedules are assigned to ATCs. It requires a minimum of 8 ATCs. The assignments are also presented in the figure. The solution results in an uneven distribution of shift schedules to ATCs. Some ATCs are assigned 2 shift schedules, some 3, some 4 and some 5. Once these results are obtained, it is possible to reassign the shift schedules so that the load variation can be reduced. One such rearrangement is shown in Figure 4. In this reassignment, every operator is assigned exactly 4 shift schedules.

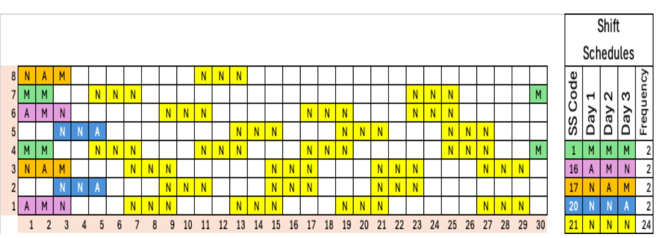

Fig. 3 Results for task 1

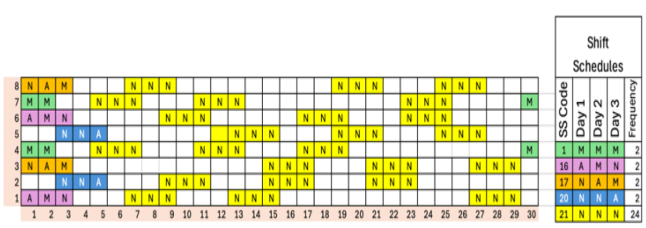

Fig. 4 Rescheduled assignment

Table 2 presents the number of shift schedules (obtained by solving OP1) and the number of ATCs required (obtained by solving OP2) for all the 9 tasks. The shift schedules are split into groups so that k^{th} the group include only those shift schedules whose starting dates belong to $\{k, k+5, k+10, k+15, k+20, k+25\}$, $k = 1,2,3,4,5$.

For example, for task 1, Group 1 consists of $\{(16,1),(16,1),(17,1), \quad (17,1),(21, \ 11),(21, \ 11),(21, 21), \ (21,21)\}$, Group 2 consists of $\{(21,7),(21,7), \ (21, 17), \ (21,17),(21,21), \ (21,21)\}$ and so on.

5.2. Results with Restricted Shift Patterns

The results of the previous subsection allow all admissible shift patterns. However, some or many shift patterns may be prohibited in actual practice. This section analyses two possible scenarios.

5.2.1. Scenario 1

It is a common practice to schedule each ATC according to 5 different patterns. MANOO, ANOOM, NOOMA, OMANO and OOMAN represent these. According to this system, any ATC will work for three consecutive (different) shifts with a two-day break between the night and the following morning shifts. This is equivalent to starting with shift pattern 6, followed by two days off, and repeating for the entire planning horizon. This problem can be directly solved by using 9 patterns for the entire planning horizon with 9 decision variables $y_1, \dots, y_9$, where y_i corresponds to the pattern $(1,0,0,0,1,0,0,0,1,0,0,0,0, \ 0,0)$ starting in i^{th} time period of the planning horizon and repeating for the rest of the periods thereafter. The minimum number of ATCs required for the 9 tasks under this system is obtained by solving this problem. According to the solution, the minimum number of ATCs required are 14, 41, 10, 24, 10, 17, 17, 10 and 10 for tasks 1 to 9 respectively.

5.2.2. Scenario 2

Under this scenario, the minimum number of shift schedules obtained by solving OP1 for any specific task is assigned to ATCs as follows. Split the shift schedules into five groups as described in Subsection 5.1. Under Scenario 2, ATCs are divided into five groups so that $Group\ k$ ATCs will be assigned to the days of the k^{th} group only. Consider task 2, for instance.

There are 24 shift schedules under $Group$ 1, and the starting days of these shift schedules are 1, 6, 11, 16, 21 and 26, with the corresponding frequencies 10, 2, 5, 2, 3 and 2, respectively. Note that these frequencies' sum equals 24 (see Table 2). Of these days, 1 is the mode, and the modal frequency is 10. As the ATCs assigned to this group work only on these days, 10 ATCs will be required to handle the 24 shift schedules of this group. Therefore, the number of ATCs required to handle all the tasks (of the five groups) equals the sum of the group modal frequencies.

Table 3. ATCs requirements under 3 different methods

	Tasks									
Method	1	2	3	4	5	6	7	8	9	Total
Scenario1	14	41	10	24	10	17	17	10	10	153
Scenario 2	12	37	9	21	9	16	16	9	9	138
Scenario 3	8	22	6	13	6	9	9	6	6	85

Table 4. Summary of results of applying OP3 to the ATC scheduling problem

	Tasks										
	1	2	3	4	5	6	7	8	9	ATCs	
Skill level	1	2	1	3	1	4	1	5	5	Used	Available
1	0	0	6	0	6	0	9	0	0	21	21
2	8	22	0	0	0	0	0	0	0	30	41
3	0	0	0	13	0	0	0	0	0	13	41
4	0	0	0	0	0	1	0	0	0	1	1
5	0	0	0	0	0	8	0	6	6	20	33
R_t	8	22	6	13	6	9	9	6	6	85	137

The group modal frequencies for task 2 are 10, 7, 7, 6 and 7. Hence, 37 ATCs will be required to handle the 108 shift schedules obtained by OP1 under Scenario 2. The number of ATCs required under Scenario 2 for the 9 tasks are 12, 37, 9, 21, 9, 16, 16, 9 and 9. Table 3 summarizes the minimum number of ATCs required for different tasks for the three scenarios. Scenario 3 is the solution obtained by OP1 and OP2. Clearly, the method proposed in this article is far superior to the solutions obtained by the other two methods as the method considers all possible options, whereas the others consider restricted options.

5.3 Scheduling ATCs using OP3

OP3 formulation is used to draw the final schedule of ATCs. ATC availabilities, the cost elements and the incidence matrix for this problem are taken from Figure 2. For the task requirements, R_t, consider the results of Scenario 3. The cost c_{it} is taken as follows: $c_{it} = 10 * (g_i - \overline{g}_t) + 10$, where g_i is the skill level of ATC i and $\overline{g}_t$ is the minimum skill level required for task t. A summary of the solution and relevant inputs is presented in Table 4. The numbers below the task numbers in the table are the minimum skill level requirements. The last two columns present the available ATCs at different skill levels and their assigned number (under the column" Used"). According to this solution, 20 of the level 5 employees are assigned tasks - 8 to tasks 6, and 6 each to tasks 8 and 9 (see second row from the bottom). The choice of costs makes better utilization of the skilled ATCs. That is, utilizing higher-skilled ATCs is discouraged to get tasks that require lower skills.

6. Scope for Future Research

It should be noted that the approach presented in this article provides near-optimal solutions. This is due to the method developed by Lalita and Murthy [16], which, while not always yielding exact optimal solutions, establishes optimality bounds to ensure that the solutions are close to optimal. Extensive numerical evidence supports this claim [16]. The main challenge in achieving an optimal solution largely depends on the outcome of the OP1 model. Suppose the solution to the problem of determining the minimum number of operators required to handle all tasks is optimal. In that case, the OP3 model will also yield an optimal solution to the problem. One potential improvement in solving the minimum operator requirement problem is to ensure that the shift schedules generated by the OP1 model are uniformly distributed among operators. This aspect warrants further investigation. An approach worth exploring is to modify the objective function of the OP1 model to promote more balanced shift allocations. The revised solution shown in Figure 4, derived from the initial solution in Figure 3, was manually adjusted to achieve this balance. However, this reallocation process could be formulated as an optimization problem, another promising future research direction. Additionally, future research could explore the integration of real-time scheduling adjustments into the proposed methodology. In dynamic environments such as air traffic control, unexpected events and fluctuations in demand can necessitate immediate schedule changes. Developing adaptive algorithms that can respond to real-time data and adjust schedules on the fly would enhance the robustness and flexibility of the scheduling system. This could involve incorporating machine learning techniques to predict and respond to real-time changes or developing hybrid models combining fixed scheduling and dynamic adjustment capabilities. While this article demonstrates the methodology primarily for air traffic controller scheduling, future research could evaluate the effectiveness of the proposed method in other fields facing similar multi-skilled scheduling challenges.

7. Conclusion

This article presents a simple and effective solution for addressing multi-skilled personnel scheduling problems, focusing on air traffic controller scheduling. By utilizing Mixed Integer Linear Programming (MILP) formulations, the proposed method provides a streamlined approach to determining staffing requirements for shift scheduling and generating monthly rosters. While the primary application of this methodology is in the context of ATC scheduling at a medium-sized airport, it can be easily adapted to a wide range of industries facing similar multi-skilled scheduling challenges. The approach builds on the work of Lalita and Murthy [16], extending their method for single-skilled operators to accommodate multi-skilled personnel, making it a versatile tool for solving complex scheduling problems. The simplicity and efficiency of the MILP-based solution offer a practical alternative to more complex optimization models typically used in the literature, enabling organizations to optimize workforce allocation while adhering to operational constraints. Large-scale problems can be solved quickly with minimal processing time because the three models, OP1, OP2, and OP3, are simple assignment problems. The proposed solution significantly contributes to multi-skilled personnel scheduling applications across diverse industries.

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